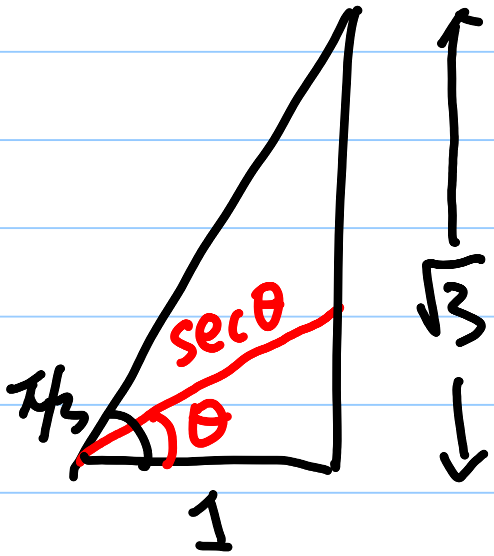
Assignment 3:

§15.4

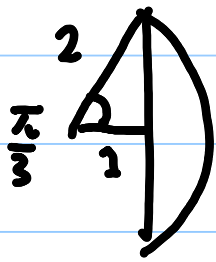
Q4:

$\{(r,\theta) \mid 0 \le r \le \sec\theta,\ 0 \le \theta \le \frac{\pi}{6}\}$



Q6:

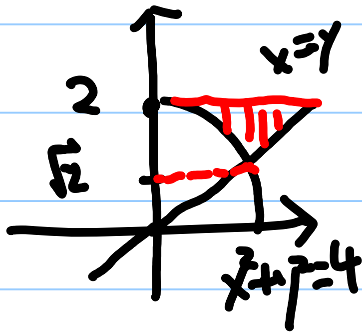
$\{(r,\theta) \mid -\frac{\pi}{3} \le \theta \le \frac{\pi}{3},\ \sec\theta \le r \le 2\}$



Q16:

$$\int_{\sqrt{2}}^{2} \int_{\sqrt{4-y^2}}^{y} dx\,dy$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{2}^{2\sec(\frac{\pi}{2}-\theta)} r\,dr\,d\theta$$



$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\left[\left(2\sec\left(\tfrac{\pi}{2}-\theta\right)\right)^2 - 2^2\right] d\theta$$

$$= 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta - 1 \, d\theta$$

$$= \left[-2\cot\theta - 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 2 - \frac{\pi}{2}$$

<u>Q19:</u>

$$\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}} \, dx\, dy$$

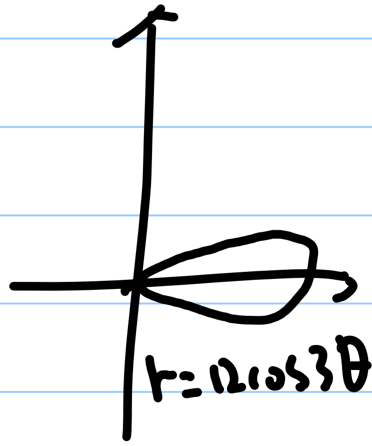
$$= \int_0^{\frac{\pi}{2}}\int_0^{\ln 2} e^r \, r \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} d\theta \cdot \int_0^{\ln 2} r \, de^r$$

$$= \frac{\pi}{2}\left[[re^r]_0^{\ln 2} - \int_0^{\ln 2} e^r dr \right]$$

$$= \frac{\pi}{2}\left[2\ln 2 - 1 \right]$$

Q29.



Area

$$= \int_{-\frac{\pi}{6}}^{\pi/6} \int_0^{12\cos 3\theta} r\,dr\,d\theta$$

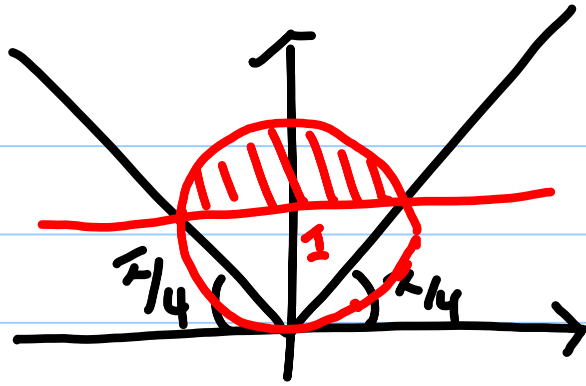
$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}(12\cos 3\theta)^2\, d\theta$$

$$= \int_{-\pi/6}^{\pi/6} 36(\cos 6\theta + 1)\, d\theta$$

$$= \left[6\sin 6\theta + 36\theta \right]_{-\pi/6}^{\pi/6}$$

$$= 12\pi$$

Q4b



Area $= \frac{1}{2}(\pi(1)^2)$

$= \frac{1}{2}\pi$

$r = 2\sin\theta$

$\Leftrightarrow \sqrt{x^2+y^2} = 2y/\sqrt{x^2+y^2}$

$\Leftrightarrow x^2+y^2 = 2y$

$\Leftrightarrow x^2+(y-1)^2 = 1$

$r = \csc\theta$

$\Leftrightarrow r = r/y$

$\Leftrightarrow y = 1$

Ex 15.5

Q20:

$$\int_0^7 \int_0^2 \int_0^{\sqrt{4-q^2}} \frac{q}{r+1}\, dp\, dq\, dr$$

$$= \int_0^7 \frac{dr}{r+1} \cdot \int_0^2 q\sqrt{4-q^2}\, dq$$

$$= \ln 8 \cdot \left[-\frac{1}{3}(4-q^2)^{3/2}\right]_0^2$$

$$= \frac{8}{3}\ln 8 = 8\ln 2$$

Q25:

$$\text{volume} = \int_0^2 \int_0^{2-z} \int_0^{4-y^2} dx\, dy\, dz$$

$$= \int_0^2 \int_0^{2-z} 4-y^2\, dy\, dz$$

$$= \int_0^2 4(2-z) - \frac{1}{3}(2-z)^3\, dz$$

$$= \left[-2(z-2)^2 + \frac{1}{12}(z-2)^4\right]_0^2$$

$$= 8 - \frac{4}{3}$$

$$= \frac{20}{3}$$

<u>Q30.</u>

$$\text{Volume} = \int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y} dz\, dy\, dx$$

$$= \int_0^2 \int_0^{4-x^2} 4-x^2-y \, dy\, dx$$

$$= \int_0^2 (4-x^2)^2 - \frac{1}{2}(4-x^2)^2 \, dx$$

$$= \int_0^2 8 - 4x^2 + \frac{1}{2}x^4 \, dx$$

$$= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5 \right]_0^2$$

$$= 16 - \frac{32}{3} + \frac{16}{5}$$

$$= \frac{128}{15}$$

Q 43

$$\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2}\, dx\, dy\, dz$$

$$= \int_0^1 \int_{\sqrt[3]{z}}^1 8\pi \cdot \frac{\sin \pi y^2}{y^2}\, dy\, dz$$

$$= \int_0^1 \int_0^{y^3} 8\pi \frac{\sin \pi y^2}{y^2}\, dz\, dy$$

$$= \int_0^1 8\pi y \sin \pi y^2\, dy$$

$$= -4 \cos \pi y^2 \Big|_0^1 = 4$$